AP Calculus AB — Worksheet 2 - Quadratic Functions — Name:

Dr. Paul L. Bailey — Monday, October 31, 2022

A *quadratic function* is a polynomial of degree two.

The *standard form* of a quadratic function is

$$f(x) = ax^2 + bx + c,$$

where a, b, and c are real numbers. The graph is a *parabola* which opens upward if $a > 0$ and opens downward if $a < 0$. The y-intercept is the point $(0, f(0))$, and we see that $f(0) = c$. The zeros of the function are the values of x such that $f(x) = 0$. The *quadratic formula* says that $f(x) = 0$ if and only if

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.}$$

The *discriminant* of the quadratic function is

$$\Delta = b^2 - 4ac;$$

this determines the number of real zeros. There are three cases:

(a) if $b^2 - 4ac > 0$, there are two real zeros;

(b) if $b^2 - 4ac = 0$, there is one real zero;

(c) if $b^2 - 4ac < 0$, there are no real zeros.

The x-intercepts (if any) are the points $(x, 0)$, where x is a real zero.

The *shifted form* of a quadratic function is

$$f(x) = a(x - h)^2 + k,$$

where a, h, and k are real numbers. The shifted form tells how the graph of $f(x)$ is obtained from the graph of x^2, as follows:

(a) shift horizontally by h;

(b) stretch vertically by $|a|$;

(c) reflect across the x-axis if a is negative;

(d) shift vertically by k.

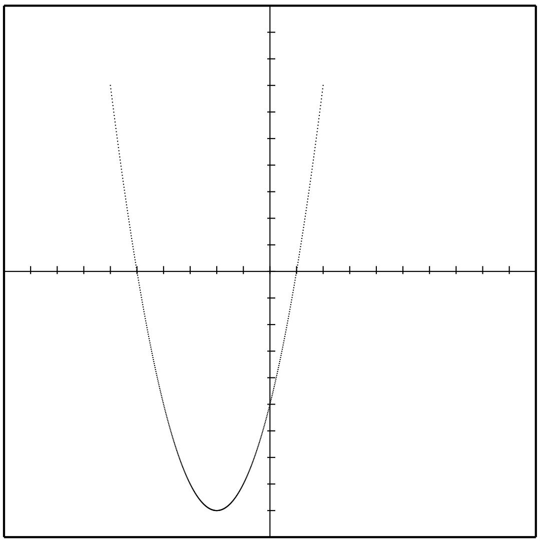
The point (h, k) where the graph turns around is called the *vertex*. Thus k is the *minimum value* of the function if $a > 0$, and is the *maximum value* of the function is $a < 0$.

We can convert from standard form to shifted form by completing the square, which leads to:

$$h = -\frac{b}{2a} \quad \text{and} \quad k = c - \frac{b^2}{4a}.$$

We can convert from shifted form to standard form by squaring and simplifying, which leads to:

$$b = -2ah \quad \text{and} \quad c = ah^2 + k.$$



Example: $f(x) = 4x - 5 + x^2$

Standard Form: $f(x) = x^2 + 4x - 5$

Shifted Form: $f(x) = (x+2)^2 - 9$

a: 1 **b:** 4 **c:** -5 **h:** -2 **k:** -9

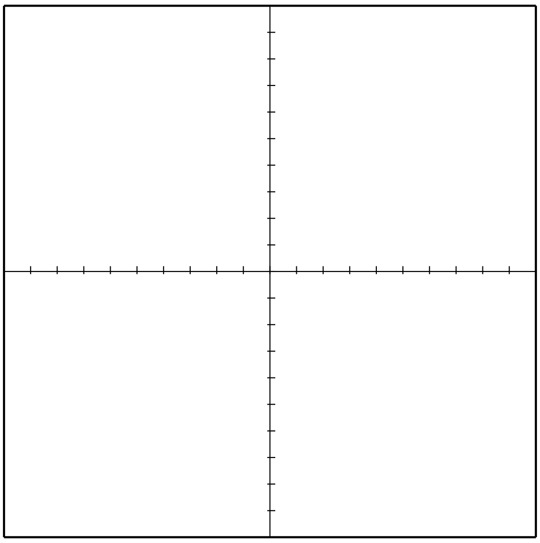
Discriminant: 36

Zeros: $x = -5$ and $x = 1$

y-intercept: $(0, -5)$

x-intercept(s): $(-5, 0)$ and $(1, 0)$

Vertex: $(-2, -9)$



Quadratic Function: $f(x) = x^2 - 6x + 8$

Standard Form:

Shifted Form:

a: **b:** **c:** **h:** **k:**

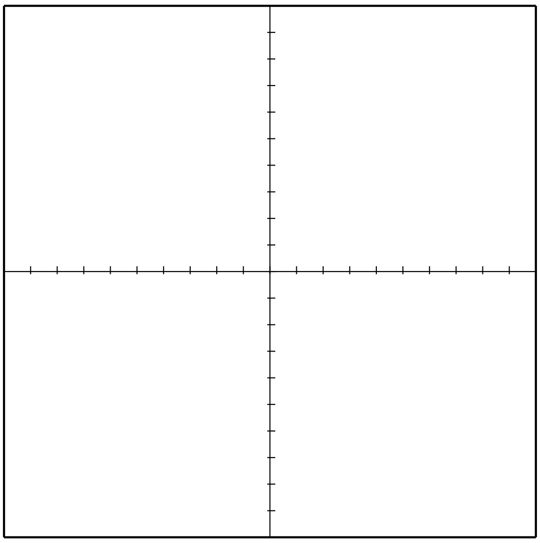
Discriminant:

Zeros:

y-intercept:

x-intercept(s):

Vertex:



Quadratic Function: $f(x) = (x+2)^2 - 5$

Standard Form:

Shifted Form:

a: **b:** **c:** **h:** **k:**

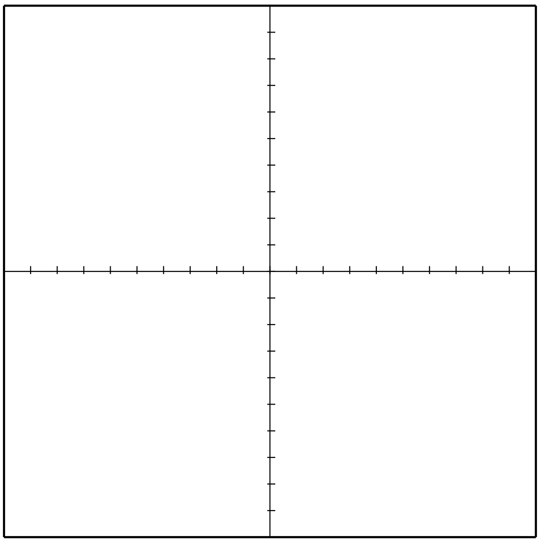
Discriminant:

Zeros:

y-intercept:

x-intercept(s):

Vertex:



Quadratic Function: $f(x) = x^2 - 6x + 9$

Standard Form:

Shifted Form:

a: **b:** **c:** **h:** **k:**

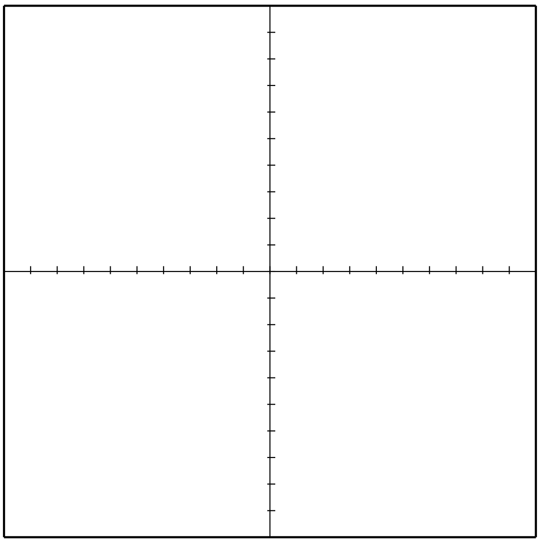
Discriminant:

Zeros:

y**-intercept:**

x**-intercept(s):**

Vertex:



Quadratic Function: $f(x) = 9 - x^2$

Standard Form:

Shifted Form:

a: **b:** **c:** **h:** **k:**

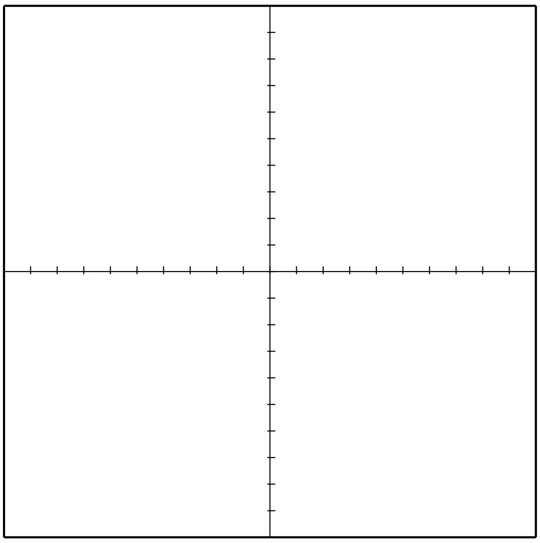
Discriminant:

Zeros:

y**-intercept:**

x**-intercept(s):**

Vertex:



Quadratic Function: $f(x) = 6x - x^2$

Standard Form:

Shifted Form:

a: **b:** **c:** **h:** **k:**

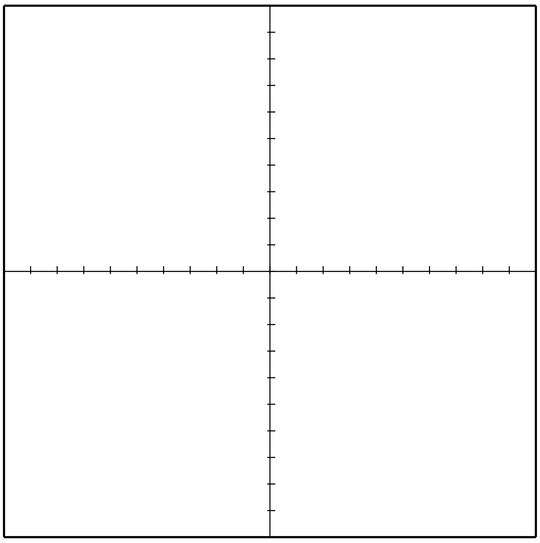
Discriminant:

Zeros:

y**-intercept:**

x**-intercept(s):**

Vertex:



Quadratic Function: $f(x) = x^2 - 5x + 2$

Standard Form:

Shifted Form:

a: **b:** **c:** **h:** **k:**

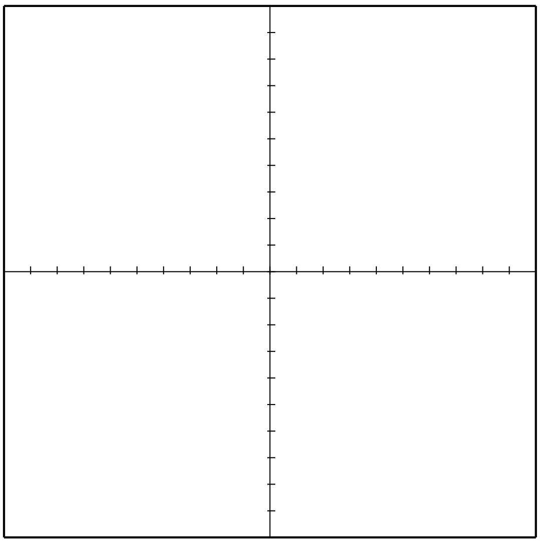
Discriminant:

Zeros:

y**-intercept:**

x**-intercept(s):**

Vertex:



Quadratic Function: $f(x) = (3x - 7)(-x + 1)$

Standard Form:

Shifted Form:

a: **b:** **c:** **h:** **k:**

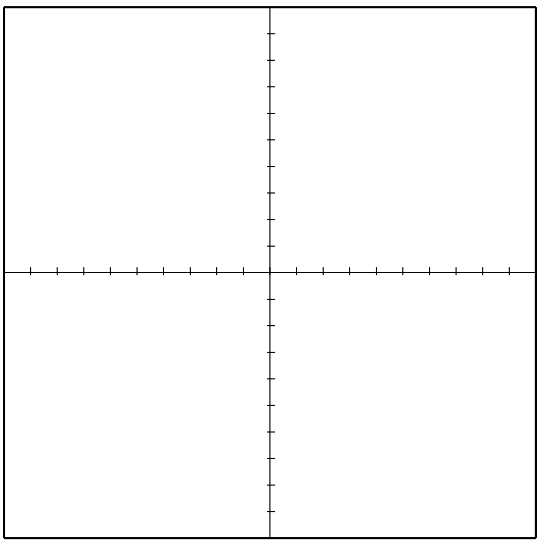
Discriminant:

Zeros:

y**-intercept:**

x**-intercept(s):**

Vertex:



Quadratic Function: $f(x) = 6 + x^2 - 4x$

Standard Form:

Shifted Form:

a: **b:** **c:** **h:** **k:**

Discriminant:

Zeros:

y**-intercept:**

x**-intercept(s):**

Vertex: